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Effect of Coherent State of Quarks and Mesons on Hadron Properties

T. S. T. Aly,[1] M. Rashdan,[2] and M. Abu-Shady[3,4]

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The effect of coherent state of quarks and mesons on hadron properties has been investigated in a chiral quark model. A new parameter set is derived. A comparison with previous work shows that the properties of the nucleon and delta are greatly improved by increasing the degree of coherent and the sigma-meson mass.

KEY WORDS: linear sigma model; coherent state.

1. INTRODUCTION

Quantum chromodynamic (OCD) is widely believed to be the correct theory of the stong interactions, Unfortunately, remarkably little progress has been made in sloving the theory in the low-energy region relevant to nuclear physics. In spite of some impressive achievements in lattice QCD, it is unrealistic to expect any major contribution from this field to nuclear physics in the difficult task of formulating phenomenological models that, while being computationally tractable, capture the essence of QCD. One of the most difficult challenges faced in pursuing this task is how to model a system that is believed to have quarks confined inside hadrons at low density but free quarks at high density. Nonrelativistaic constituent quarks models have proved to be very successful in the understanding of meson and baryon spectroscopy (Isgur, 1986). on the other side of the spectrum are the the field theoretical models that incorporate Lorentz invariance and chiral symmetry right from the outset. Some of these modele have, in fact been used with either quarks or nucleons as fundamental fermionic degrees of freedom. In the standard σ-model of Gell-Mann and Levy (1960) the small empirical values of the pion

[1] Faculty of Science, Helwan University, Egypt.
[2] Department of Mathematics and Theoretical Physics, Atomic Energy Authority, Cairo, Egypt.
[3] Faculty of Science, Menofia University, Egypt.
[4] To whom correspondence should be addressed at Faculty of Science, Menofia University, Egypt; e-mail: abu_shady_1999@yahoo.com.

mass and of the pion nucleon *s*-wave amplitude are a direct consequence of chiral symmetry.

Recently, the one meson exchange potential (OMEP) (Aly, 2004; Glozman and Riska, 1996; Glozman *et al.*, 1998) is introduced to the nonrelativistic constituent quark model (NRCQM) instead of using the one gluon exchange potential (OGEP) (Ishur and Karl, 1978). This meson belongs to the pseudoscalar octet, and is closely related with the the spontaneous breaking of chiral symmetry. The NR-CQM with the OMEP has succeeded in reproducing the observed baryon spectra (Particle Data Group *et al.*, 1998). However, the meson in this model is treated on the basic of a perturbative idea in spite of the strong meson-quark coupling. Furthermore, the static approximation is used in the OMEP, and dynamical mesonic effects such as a self energy are suppressed (Kowata *et al.*, 1998).

Goeke *et al.* (1988) investigated hadron properties in a chiral model for the nucleon based on the linear sigma model with scalar-isoscalar and scalar-isovector mesons coupled to quarks and they using the coherent pair approximation. That work has been reexamined by Aly *et al.* (1999) and they corrected some misprints of the work of Goeke *et al.* (1988). However, the coherent parameter has been fixed in the these work to $x = 1$. Although the coherent approximation is expected to be more better than the mean field approximation since some quantum effects are considered but the results are not qualitatively improved (Aly *et al.*, 1999). For example the relative error in the magnetic moment of the proton is about 40% and in the neutron radius about 97%. The aim of this work is to investigate the effect of the degree of coherent on the nucleon properties. A new parameter set has also been derived. We used the sigma model Lagrangian of Gell-Mann and Levy which describes explicit quark interact with scalar-isoscalar sigma meson and pseudoscalar-isovector (pion, $\vec{\pi}$) degrees of freedom in the coherent pair approximation.

2. THEORETICAL DESCRIPTION

The chiral Lagrangian of Gell-Mann and Levy (1960) with explicit quark degrees of freedom, after chiral symmetry breaking inducing a pion mass can be written as:

$$L(x) = i\overline{\Psi}\partial_\mu\gamma^\mu\hat{\Psi} + \frac{1}{2}(\partial_\mu\hat{\sigma}\partial^\mu\hat{\sigma} + \partial_\mu\vec{\pi}\cdot\partial\mu\vec{\pi}) + g\overline{\Psi}(\hat{\sigma} + i\gamma_5\vec{\tau}\,.\,\vec{\pi})\hat{\Psi} - U(\hat{\sigma},\vec{\pi}) \quad (1)$$

with

$$U(\hat{\sigma},\vec{\pi}) = \frac{\lambda^2}{4}\mathbf{i}\hat{\sigma}^2 + \hat{\pi}^2 - \nu^2 \not{c}_2 - f_\pi m_\pi^2\hat{\sigma} \quad (2)$$

where f_π is the pion decay constant, m_π is the pion mass, and ν, g and λ are constants to be determined. The quark, sigma and $\vec{\pi}$- mesons are quantum fields

Table I. The Coherent Parameter, Pion-nucleon Coupling Constant and the Sigma Meson Mass in a Comparison with that of Aly *et al.* (1999)

Quantity	This work	Aly and *et al.* (1999)
x	3	1
g	5.2	5
m_σ(GeV)	1.2	0.7

denoted by (ˆ). Spontaneous symmetry breaking generates masses for the quark and sigma and the linear sigma term, which breaks the chiral symmetry and generates the small pion mass which would be zero otherwise as the Goldstone boson of the theory. The vacuum then has non-vanishing field expectation value,

$$\frac{\partial U}{\partial \vec{\pi}} = 0 \Longrightarrow \vec{\pi}_0 = 0, \quad \frac{\partial U}{\partial \hat{\partial}} = 0 \Longrightarrow \sigma_0 = f_\pi. \tag{3}$$

Then the three undetermined constants in the original Lagrangian can be written in terms of the three effective masses: $m_q = -gf_\pi, m_\sigma^2 = \lambda^2(3f_\pi^2 - \nu^2)$, and $m_\pi^2 = \lambda^2(f_\pi^2 - \nu^2)$. We take the experimental values $f_\pi = 93$ MeV and $m_\pi = 139.6$ MeV, leaving g and m_σ as well as the coherent parameter x as free parameters which must be determined. The additional parameters are constrained by minimization. Introducing the conjugate momenta, one write the Hamiltonian density as

$$\begin{aligned}\hat{H}(r) = {} & \frac{1}{2}\{\hat{P}_\sigma(r)^2 + [\mathbf{3}\nabla\hat{\sigma}(r)]^2 + \hat{P}_\pi(r)^2 + \mathbf{h}\nabla^{\mathbf{b}}\pi(r)^{\mathbf{i}_2}\} \\ & + U(\hat{\sigma}, \vec{\pi}) + \hat{\Psi}^\dagger(r)^{\mathbf{3}} - i\alpha\vec{\nabla}\hat{\Psi}(r) \\ & - g(r)\hat{\Psi}^\dagger(r)\beta\hat{\sigma}(r) + i\beta\gamma_5\vec{r}.\vec{\pi}\,\hat{\Psi}(r),\end{aligned} \tag{4}$$

Table II. The Energy Contributions (in MeV) to Nucleon and Delta when Using $g = 5.2$, $m_\sigma = 1200$ MeV and the Coherent Parameter $x = 3$

Quantity	Nucleon	Delta	Nucleon (Aly *et al.*, 1999)	Delta (Aly *et al.*, 1999)
Quark kinetic energy	991.8	926.1	1124	975
Sigma kinetic energy	350.4	325.9	304	268
Pion kinetic energy	494.6	474.8	236	185
Quark-meson interaction	−754.3	−493.4	−675	−318
Meson interaction energy	−26.5	−16.9	84	114
Baryon mass	1055.9	1216	1073	1224
Nucleon-Delta mass difference	160.1		140	

Table III. Nucleon Observables Using $x = 3$, $m_\sigma = 1200$ MeV and $g = 5.2$

Quantity	Quark	Meson	Total	Quark (Aly *et al.*, 1999)	Meson (Aly *et al.*, 1999)	Total (Aly *et al.*, 1999)	Expt.
r_c^2 (proton) (fm^2)	0.63	0.066	0.7	0.533	0.023	0.556	0.7
r_c^2 (neutron) (fm^2)	0.03	−0.06	−0.03	0.019	−0.023	−0.004	−0.12
Magnetic moment (proton)	1.55	0.377	1.93	1.53	0.18	1.71	2.79
Magnetic moment (neutron)	−1.19	−0.38	−1.6	−1.13	−0.18	−1.31	−1.91

where α and β are the usual Dirac matrices. In the above expression $\hat{\Psi}$, $\hat{\sigma}$, and $\mathbf{b}\pi$ are quantized field operators with the appropriate static angular momentum expansion (Aly *et al.*, 1999). The Fock state for the nucleon is taken to be (Aly *et al.*, 1999)

$$[N \cdot T_3 J_z\rangle = \mathbf{h}\alpha^{\mathbf{i}}|n\rangle \otimes P_0^{0®} \not{c} T_3 J_z + \beta\mathbf{i}|n\rangle \otimes P_1^{1®} \not{c} T_3 J_z$$
$$+ \gamma \mathbf{3}\delta\mathbf{m} \otimes P_1^{1®} T_3 J_z \mathbf{i} \times \mathbf{E}, \quad (5)$$

where-$^{®}|\mathbf{P}\rangle^{®}$ $\not{c}$ the coherent sigma field state with the property: $\langle\mathbf{P}|\hat{\sigma}(r)|\mathbf{P}\rangle = \sigma(r)$, and $P_0^0 \mathbf{i} P_{1m}^{1w}$ are pion coherent-pair states to be determined. The normalization of the nucleon state requires $\alpha^2 + \beta^2 + \gamma^2 = 1$. (for the details we refer the reader to Aly *et al.*, 1999; Goeke *et al.*, 1988).

3. RESULTS AND DISCUSSION

The nucleon observables are calculated the field equations of the sigma model described in Section 2 in the coherent pair approximation (Aly *et al.*, 1999; Goeke *et al.*, 1988). We found a new set of parameters which better describes the observables. These parameters are listed in Table I in a comparison of that of Aly *et al.* (1999). The results of the energy contributions of nucleon and delta are listed in Table II and for the nucleon and delta observables are listed in Table III. As seen from these tables the present set of model parameters successfully described nucleon properties in comparison with Aly *et al.* (1999). All observables are greatly modified. This is mainly due to the increase in the coherent parameter x which on the other hand increases the mesonic contributions. This could also indicate to the important of mesonic interactions which in quark-meson theories simulate gluonic interactions. On the other hand, this could suggest to increase mesonic contributions by adding other mesons in the linear sigma model such as the omega meson, which is important for the hard-core interaction of the nucleon-nucleon potential.

REFERENCES

Aly, T. S. T. (8 June 2004). *Chaos, Solitons and Fractals* (accepted).
Aly, T. S. T., McNeil, J. A., and Pruess, S. (1999). *Physical Review D* **60**, 1114022.
Gell-Mann, M. and Levy, M. (1960). *Nuono Cinmento* **16**, 705.
Glozman, L. Y. and Riska, D. O. (1996). *Physics Reports* **268**, 263.
Glozman, L. Y., Plessas, W., Varga, K., and Wagenbtunn, R. F. (1998). *Physics Review D* **58**, 094030.
Goeke, K., Harve, M., Gru, F., and Urbano, J. N. (1988). *Physical Review D* **37**, 754.
Isgur, N. (1986). In *Proceedings of the relativistic Dynamics and Qurak-Nuclear Physics*, edited by M. B. Johnson and A. Picklesimer, Wiley, New York, p. 619.
Ishur, N. and Karl, G. (1978). *Physical Review D* **18**, 4187.
Kowata, H., Arima, M., and Masutani, K. (1998). *Progress of Theoretical Physics* **100**, 339.
Particle Data Group, Caso, C. *et al.* (1998). *European Physical Journal C* **3**, 1.